Exercises for Chapter 2

1. In each of the four cases given, find a function $y(t)$ that solves the following equation:

   (a) $\frac{dy}{dt} = 5y$

   (b) $\frac{dy}{dt} = -3y$

   (c) $\frac{dy}{dt} = 12y$

   (d) $\frac{dy}{dt} = -1.5y$

2. In each of the four cases in Exercise 1, find a solution with

   (a) $y(0) = 1$

   (b) $y(1) = 1$

   (c) $y(-1) = 1$

   (d) $y(-1) = -1$

3. Which of the four solutions to Exercise 2a is largest when $t = 10$? Which is smallest when $t = 10$.

4. Find the first-order Taylor's approximation at $x_0 = 0$ for the functions

   (a) $f(x) = \sin x$

   (b) $f(x) = e^x$

   (c) $f(x) = x/(1 + x^2)$

   (d) $f(x) = e^x \sin$

   (e) $f(x) = \sin(e^x)$

   [Remember that the first-order Taylor's expansion for a function $f(x)$ at a point x_0 is defined to be the simplest function, $g(x)$, whose value at x_0 is the same as that of f and whose first derivative at x_0 is also the same as f's first derivative at x_0.]

5. For a certain microorganism, birth is by budding off a fully formed copy of itself. Suppose that under reasonable favorable laboratory conditions (plenty of food and no predation), such births occur on average four times per day, and an individual lives, on the average, one day. Write a differential equation for the population, $p(t)$, of the microorganism as a function of time. Then find the solution given that at time zero, the population numbered 1000.

6. Design a hypothetical experiment with snails that would verify or rule out each of the two assumptions behind the simple model I present for snail handedness in the commentary on Reading 2.2.