Space occupation by tree crowns obeys fractals laws: evidence from 3D digitized plants

C. Godin[1], O. Puech[2], F. Boudon[2], H. Sinoquet[3]

[1] INRIA, UMR Botanique et Bio-informatique des Plantes, Montpellier
[2] INRA, UMR Botanique et Bio-informatique des Plantes, Montpellier
[3] INRA, UMR PIAF, Clermont-Ferrand

Introduction

Plant geometry is a key factor of the modelling of plant eco-physiological interaction with the environment. This interaction may concern either the abiotic (resource capture, heat dissipation) or the biotic (disease propagation, insect movement) environment. Depending on applications, plant geometry has been abstracted in various ways (Godin, 2000): simple volumic shapes (like ellipsoids, cones, or "big leaves" used in turbib medium approaches) or detailed models to render realistic trees. Global descriptions are simple and contain few parameters. However, they do not capture the irregular nature of plant shapes which severely limits the generalization capacity of the model. On the other hand, detailed descriptions tentatively address this problem but require over-parameterization of geometry, leading to non-parsimonious models. Characterizing the irregularity of plant shapes with a few parameters is thus a challenging problem in the eco-physiological modelling of plants.

Fractal geometry was introduced as a new conceptual framework to analyze and model the nature of irregular shapes (Mandelbrot, 1983). This framework has been applied in different occasions to the modelling of plant structure. Generative approaches used fractal concepts to illustrate how intricate vegetal-like structures could be generated using parsimonious models of plants shapes (Smith, 1984), (Barnsley, 1988), (Prusinkiewicz and Hanan, 1989). Such models were used to generate artificial plants in modelling applications, *e.g.* (Chen *et al.*, 1994), (Prusinkiewicz *et al.*, 2001). Fractal geometry was used also to analyse the irregularity of plants by determining their supposed *fractal dimension*. This parameter is of major importance in the study of irregularity since it characterizes the way plants physically penetrate into the 3D space. Most of these studies were carried out on woody structures, and especially on root systems (Fitter, 1987), (Eshel, 1998), (Oppelt *et al.*, 2000). A few works have indirectly addressed the problem of determining the fractal dimension of plant canopies. Relying on the assumption that plants are self-similar organisms (Zeide and Pfeifer, 1991) (Zeide, 1991) used a comparison between estimated leaf areas and crown surface to estimate the fractal dimension of forest trees. Fractal dimension was also estimated from 2D photographs of crowns (Critten, 1997), (Morse *et al.*, 1985). However, such a technique always under-estimate the actual fractal dimension (Falconer, 1990), and is not accurate. These studies provided reasons to think that plant crowns have fractal properties, but this was not yet proved directly from the 3D analysis of the plant shape. This study aims at showing fractal properties of plants by using recent techniques developed for crown 3D digitizing at leaf resolution (Sinoquet *et al.*, 1998). For this purpose, a range of fractal methods is applied to various 3D digitized tree databases and to theoretical plants generated from fractal rules.

Material and method

3D Plant database

Eight 3D plants were included in the study. Four real trees were 3D-digitised in the field, while 4 additional plants were generated from theoretical assumptions.

<u>Digitized plants</u>

One three-year old hybrid walnut tree (NG38 x RA) and two two-year old mango trees (cv. Nam Nok Mai) were 3D-digitised at leaf scale, according to (Sinoquet *et al.*, 1998), in August 1998 and November 1997, respectively. The walnut tree was grown in an experimental plot in Clermont-Ferrand INRA research centre, France, while the mango trees were grown in a commercial farm in Bangbun, 150 km South-East from Bangkok, Thailand. The location and orientation of each leaf was recorded with a magnetic digitiser (Fastrak 3Space, Polhemus, Vermont) while leaf length and width were measured with a ruler. A sample of leaves was harvested on similar trees to establish an allometric relationship between individual leaf area and the product of leaf length and width. Individual area of sampled leaves was measured with a leaf area meter Li-cor 3100. The data sets therefore consisted of a collection of leaves, the size, the orientation and the location of which have been measured in the field.

Four four-year peach trees (cv. August Red) were digitised in May 2001 in CTIFL Center, Nîmes, South of France, at current-year shoot scale, one month after bud break. Given the high number of leaves (≈14,000), digitising at leaf scale was impossible. The magnetic digitising device was therefore used to record the spatial co-ordinates of the bottom and top of each leafy shoot. Thirty shoots were digitised at leaf scale in order to derive i) leaf angle distribution, ii) allometric relationships between number of leaves, shoot leaf area and shoot length. Leaves of each shoot were then generated from i) allometric relationships, ii) sampling in leaf angle distribution and iii) additional assumptions for the internode length and the distribution of leaf size within a shoot.

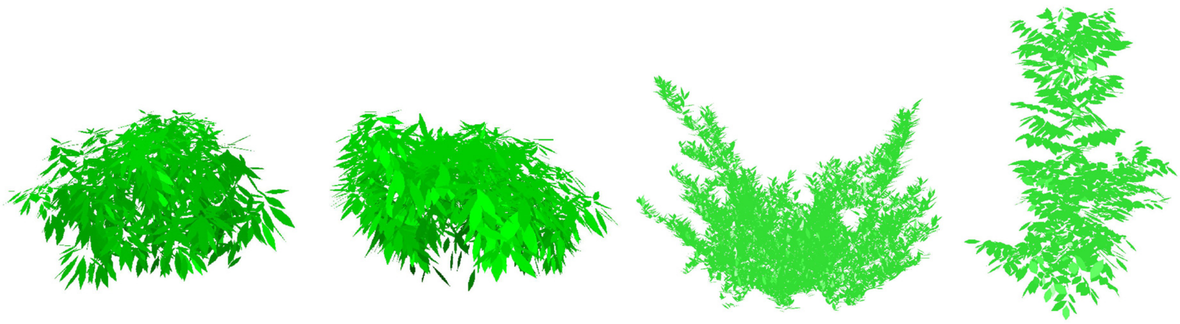

Figure 1. Four digitized plants. From left to right: mango tree 1, mango tree 2, peach tree, walnut tree

Theoretical plants

Three fractal plants were generated from 3D iterated function systems (Barnsley, 1988) as illustrated in Figure 2. The initial object was a tapered ellipsoid and the IFS transformation was made of *n* duplications of a contracted object by a factor *c*. If the duplications of the IFS do not overlap, the theoretical fractal dimension of the IFS attractor is:

$$D_T = \frac{\text{Ln } n}{\text{Ln } c}$$

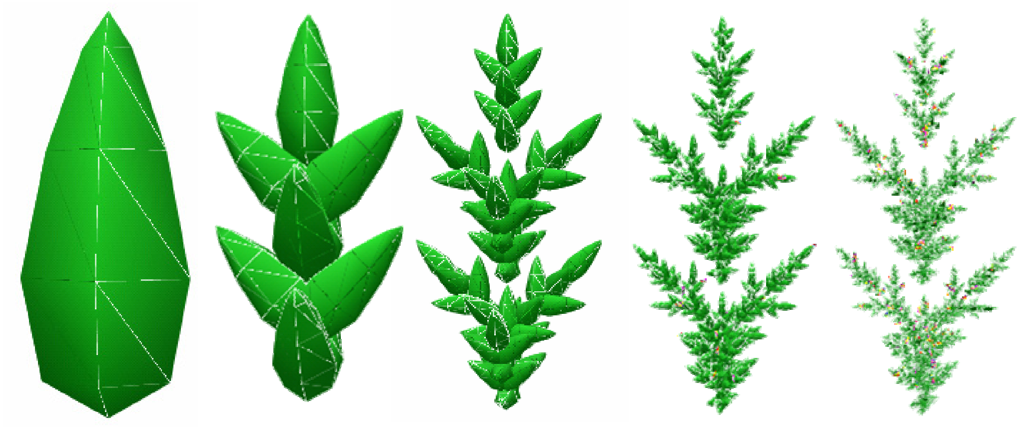

Figure 2. Series of iterations that generate a self-similar plant from an IFS consisting of 9 duplications of an initial object contracted by a factor 3 (theoretical fractal dimension D_T=2).

Three self-similar artificial canopies were generated using different IFS (respectively shown in Figure 3) : AC1 (n=9,c=3) example of Figure 2, AC2 (n=9,c=3) identical to the previous one with larger gaps between duplications, AC3 (n=7,c=3) identical to AC1 with less duplications. Each IFS was developed over 5 iterations. In addition to these self-similar plants a reference random isolated canopy (RP) was generated by randomly locating 1,000 leaves within a 1-m^3 cube. Virtual leaves were horizontal disks, the diameter of which was 10 cm.

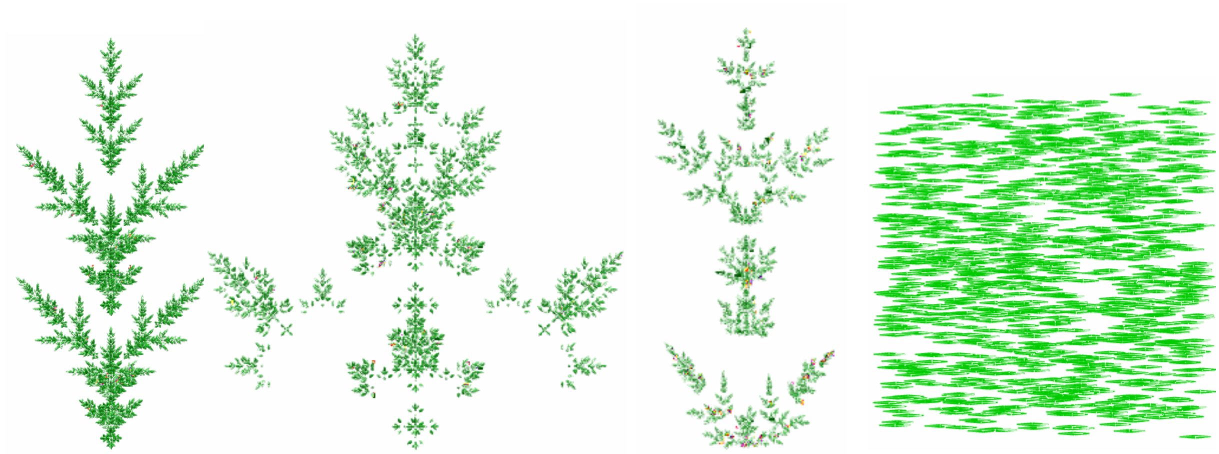

Figure 3. Four theoretical plant crowns. From left to right: fractal plants AC1, AC2, AC3, RC, with theoretical fractal dimension D_T respectively 2.0, 2.0, 1.771, and a uniform random distribution of leaves.

Methods for computing fractal properties

Different types of measures have been applied on the plant database to tentatively characterize the geometric irregularity of plants.

Fractal dimension

Fractal dimension expresses the (constant) rate at which new geometrical details appear as one zooms in an object. In this respect, the fractal dimension is a measure of the geometric irregularity of the object. Several estimators have been developed to compute this characteristic from raw data. These estimators were usually applied on 2D images, including in application to plants canopies, leading to inaccurate estimations. In this paper, we developed 3D methods to design these estimators directly from 3D architectural data.

Box counting method. This method has been extensively used to estimate fractal dimension of objects embedded in the plane. Its adaptation to 3-D analysis consists of building a 3-D grid dividing space in voxels of size δ (volume δ^3) and counting the number $N(\delta)$ of grid voxels intercepted by the studied object at scale δ. The estimator of the fractal dimension D_b of the object is defined as:

$$D_b = \lim_{\delta \to 0} \frac{\mathrm{Ln}\, N(\delta)}{\mathrm{Ln}\, \frac{1}{\delta}}$$

The geometric scenes representing the plant crowns were designed using the PlantGL library (Boudon *et al.*, 2001). All the geometric objects were approximated by triangle meshes. To detect intersection of grid voxels with the scene objects, the algorithm described in (Françon *et al.*, 1997) was used.

The *local dimension method* (*mass method*) defines a local estimator of the fractal dimension (e.g. (Gouyet, 1992)). It relies on the observation that the "mass" $M(x,\delta)$ of the part of an object contained in a ball of varying radius δ, centered on a point x of the object, varies as:

$$M(x,\delta) = A(x,\delta).\delta^{D_l}$$

where $A(x,\delta)$ is a prefactor that is independent of δ for fractal objects as δ tends to 0. The fractal dimension can then be estimated as the average value of D_l over all points x.

The *two-surface method* is an indirect method for estimating the fractal dimension of an object. It has been used in the context of plant crown analysis by (Zeide and Pfeifer, 1991). The method compares the total leaf area S_F of supposed fractal dimension D_s to the convex envelop area S_E of the crown of dimension 2 of a plant. It can be shown that the two values are linked by the following equation (Godin, 2003):

$$S_F = k.(S_E)^{\frac{D_s}{2}}$$

where k is a constant. According to this equation, the estimation of D_s can be made from the measurement of pairs of values (S_F, S_E) on different individuals of the same species. The two-surface fractal dimension D_s is defined as the slope of the regression between the log values of these variables.

Lacunarity

The fractal dimension does not characterize completely the geometric properties of fractal objects. For instance, plants AC1 and AC2 have identical fractal dimension 2 but show geometries with different gap structures. Lacunarity has been introduced as a complementary measure to reveal such characteristics. The paper will compare different definitions of this quantity.

The most widely used definition of lacunarity corresponds to *the relative moment of order 2* of the distribution of local mass at scale δ when x varies inside the object bounding box (*i.e.* not only on the object).

$$L(\delta) = \frac{\mathbf{E}(M(x,\delta)^2)}{\mathbf{E}(M(x,\delta))^2}$$

This definition shows nice formal properties and can be computed using a gliding box algorithm (Allain and Cloitre, 1991). Alternatively, a modified definition of the lacunarity $L^{\circ}(\delta)$ can be defined where x varies only on the object, e.g. (Gouyet, 1992).

Sketch of results and discussion

Results will show the different fractal characteristics (fractal dimensions and lacunarities) computed on both real and artificial crowns. Figure 4 illustrates typical estimations of these values for the peach tree and compared to equivalent characteristics for a non-fractal random canopy.

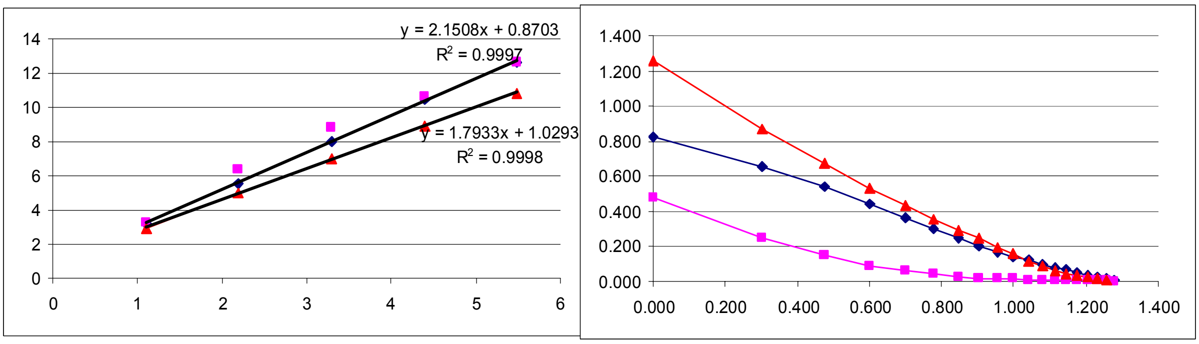

Fig 4. Comparison of Peach tree, self-similar tree AC3 and random tree crown (diamonds = peach tree, triangles = AC3, squares = random curve). Left: logarithm of number of intercepted voxels as a function of logarithm of voxel size. Right: Lacunarity.

This study shows that plants crown geometry exhibits a fractal behaviour. Theoretical plants were used to assess the quality of fractal parameter estimation on geometries for which these quantities could be computed analytically. Lacunarity is shown to be a useful complementary information of the standard fractal dimension in order to characterize plant crown irregularity. Interestingly, the fractal dimension of plants we investigated in this study is closed to 2, e.g. the dimension of a surface. Turbid medium does not behave as a fractal object (as previously reported in (Plotnick *et al.*, 1996)). The different methods to compute fractal dimension and lacunarity will be compared and their ability to characterize irregular geometry of plant crows will be discussed.

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