Sobolev orthogonal polynomials in several variables of hight order defined on product domains

Herbert Dueñas Ruiz
Omar Salazar Morales
Universidad Nacional de Colombia

Let $w_i(x)$, for every $i = 1, 2, \ldots, d$, a non negative weight function defined in the interval (a_i, b_i). Let W the weight function,

$$W(\vec{x}) = w_1(x_1)w_2(x_2)\cdots w_d(x_d),$$

$$\vec{x} = (x_1, x_2, \ldots, x_d) \in \Omega = (a_1, b_1) \times (a_2, b_2) \times \cdots \times (a_d, b_d).$$

We study the polynomials of several variables, orthogonal with respect to the inner product:

$$(f, g)_S = \int_\Omega \nabla^k f(\vec{x}) \cdot \nabla^k g(\vec{x}) W(\vec{x}) d\vec{x} + \lambda f(\vec{p}) g(\vec{p}), \tag{1}$$

where $\lambda > 0$, and $\vec{p} = (p_1, p_2, \ldots, p_d)$ in a corner of Ω. Two examples are presented using Laguerre and Gegenbauer polynomials.