IX Jaen Conference on Approximation
Úbeda, July 8th – July 13th, 2018

MARCINCKIEWICZ INEQUALITIES WITH EXPONENTIAL WEIGHTS

G. MASTROIANNI AND I. NOTARANGELO

Abstract

In 1936 J. Marcinkiewicz proved that, for any trigonometric polynomial of degree at most m, T_m, and for $1 < p < \infty$,

$$\left(\int_0^{2\pi} |T_m(x)|^p \, \mathrm{d}x\right)^{1/p} \sim \left(\frac{2\pi}{2m+1} \sum_{k=0}^{2m+1} \left|T_m\left(\frac{2k\pi}{2m+1}\right)\right|^p\right)^{1/p}$$

where the constants in "$\sim$" depend only on p. As is well known, these inequalities provide a important tool for the discretization of the L^p norm and are widely used in the study of the convergence properties of interpolation processes [5].

The case of algebraic polynomials is more difficult and the first results in this direction have been obtained by R. Askey in 1973 (see [4] or [1] and the reference therein). In fact, while the direct Marcinkiewicz-type inequality

$$\left(\sum_{k=1}^{m} \Delta x_k \left|P_{lm}\left(x_k\right) u(x_k)\right|^p\right)^{1/p} \le \mathcal{C} \left(\int_{-1}^{1} |P_{lm}(x) u(x)|^p \, \mathrm{d}x\right)^{1/p}, \quad 1 \le p < \infty\,,$$

holds with $\mathcal{C}$ depending only on p for any polynomial P_{lm} of degree lm (l fixed integer), where $\Delta x_k = x_{k+1} - x_k$, x_k is an arbitrary arcsin distributed system of nodes and u is a doubling weight [3], the converse inequality requires more restrictive assumptions. For instance, letting w and u be Jacobi weights, denoting by x_k the zeros of the mth orthonormal polynomial w.r.t. w, for any algebraic polynomial of degree at most $m-1$, P_{m-1}, and for $1 < p < \infty$, the converse Marcinkiewicz-type inequality

$$\left(\int_{-1}^{1} |P_{m-1}(x) u(x)|^p \, \mathrm{d}x\right)^{1/p} \le \mathcal{C} \left(\sum_{k=1}^{m} \Delta x_k \left|P_{m-1}\left(x_k\right) u(x_k)\right|^p\right)^{1/p}$$

holds with $\mathcal{C}$ depending only on p, if and only if

$$\frac{u}{\sqrt{w\varphi}} \in L^p \quad \text{and} \quad \frac{\sqrt{w\varphi}}{u} \in L^q\,,$$

where $\varphi(x) = \sqrt{1-x^2}$ and $\frac{1}{p} + \frac{1}{q} = 1$ [2].

In this talk, we discuss how these inequalities can be extended in the case of exponential weights on bounded or unbounded intervals of the real line.

Keywords: Marcinkiewicz-Zygmund inequalities, orthogonal polynomials, exponential weights.

AMS Classification: 41A17, 41A05, 33C47.

Bibliography

[1] **D.S. Lubinsky**, Marcinkiewicz–Zygmund inequalities: Methods and results, In: G.V. Milovanović (eds) *Recent Progress in Inequalities*, Mathematics and Its Applications, vol. 430. Springer, Dordrecht (1998), pp. 213–240.

[2] **G. Mastroianni and M.G. Russo**, Lagrange interpolation in weighted Besov spaces, *Constr. Approx.* **15** (1999) 257–289.

[3] **G. Mastroianni and V. Totik**, Weighted polynomial inequalities with doubling and A_∞ weights, *Constr. Approx.* **16** (2000) 37–71.

[4] **P. Nevai**, Géza Freud, orthogonal polynomials and Christoffel functions. A case study *J. Approx. Theory*, **48** (1986) 3–167.

[5] **A. Zygmund**, *Trigonometric series*, (2nd ed.), Vols. I–II, Cambridge University Press, New York, 1959.

Giuseppe Mastroianni,
University of Basilicata
Department of Mathematics, Computer Sciences and Economics
via dell'Ateneo Lucano 10,
85100 Potenza (Italy).
giuseppe.mastroianni@unibas.it

Incoronata Notarangelo,
University of Basilicata
Department of Mathematics, Computer Sciences and Economics
via dell'Ateneo Lucano 10,
85100 Potenza (Italy).
incoronata.notarangelo@unibas.it