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INVERSION FORMULA AND RANGE CONDITIONS FOR A VECTOR MULTI-INTERVAL FINITE HILBERT TRANSFORM IN L^2

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Abstract

Given n disjoint intervals I_j on $\mathbb{R}$ together with n functions $\psi_j \in L^2(I_j)$, $j = 1, \dots n$, and an $n \times n$ matrix Θ, the problem is to find n functions $\psi_j \in L^2(I_j)$ satisfying the system

$$\sum_{k=1}^{n} \Theta_{jk} \mathcal{H}_k \varphi_k(z) = \psi_j(z), \quad z \in I_j, \quad j = 1, \dots, n,$$

where $I = \cup_{j=1}^{n} I_j$ and $\mathcal{H}_k : L^2(I_j) \to L^2(I)$ is a finite Hilbert transform.

Since we can interpret the right hand side as a generalized vector multi-interval finite Hilbert transform, we call the formula for the solution as "the inversion formula" and the necessary and sufficient conditions for the existence of a solution as the "range conditions". In this talk we derive the explicit inversion formula and the range conditions for the case of positive definite symmetric matrix Θ in terms of the solution of the associated matrix Riemann-Hilbert Problem. We also discuss other cases of the matrix Θ.

Keywords: finite Hilbert transform, matrix Riemann-Hilbert Problem, range conditions.

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