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ON WEIGHTED NORM BOUNDEDNESS OF THE BERNSTEIN-CHLODOVSKY OPERATORS

THEODORE KILGORE, JOZSEF SZABADOS

Abstract

In 1937, I. Clodovsky used the Bernstein polynomial operators $B_n(f;x)$ to devise polynomial operators $C_n(f;x)$ which could be used to approximate continuous functions defined on $[0,\infty)$. Recently, T. Kilgore found conditions which guarantee that for a very broad class of weight functions $W(x)$ the Chlodovsky operators will converge uniformly in $C_W[0,\infty)$, the space of all continuous functions f such that $W(x)f(x) \to 0$ as $x \to 0$, to any uniformly continuous and bounded function. And further, since such functions are dense in $C_W[0,\infty)$, the Weierstrass Theorem follows, too. Essentially, the only conditions imposed upon the weight function W were that $W(x)$ must be continuous and must decay more rapidly than e^{x^α} for some $\alpha > 1$ as $x \to \infty$.

Here, the question of the uniform boundedness of $\|W(x)C_n(f;x)\|$ will be explored, under the condition that $W(x) = e^{-x^\alpha}$ and $|f(x)| \leq e^{\lambda x^\alpha}$, with $0 < \lambda \leq 1$.

Keywords: Weighted approximation, Bernstein polynomials, Chlodovsky operators.
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Theodore Kilgore,
Department of Mathematics and Statistics,

Auburn University, AL 36849 USA.
kilgota@auburn.edu

Jozsef Szabados,
Alfred Rényi Institute of Mathematics,
Reltanoda utca 13-15, H-1053, Budapest, Hungary.
szabados@renyi.hu