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ON THE NUMERICAL SOLUTION OF INTEGRAL EQUATIONS WITH FIXED SINGULARITIES OF MELLIN TYPE IN WEIGHTED UNIFORM SPACES

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Abstract

This talk deals with the numerical solution of integral equations of the following type

$$f(y) + \int_0^1 k(x,y)f(x)dx + \int_0^1 h(x,y)f(x)dx = g(y), \quad y \in (0,1], \tag{1}$$

where

$$k(x,y) = \frac{1}{x}\tilde{k}\left(\frac{y}{x}\right) \tag{2}$$

is a Mellin kernel with $\tilde{k}$ a given function on $[0,+\infty)$ satisfying proper assumptions, $h(x,y)$ and $g(y)$ are given sufficiently smooth functions and $f(y)$ is the unknown.

Many problems in mathematics, physics and engineering give rise to such kind of integral equations. For example they occur when boundary integral methods are used on domains with corners.
They are characterized by the presence of a Mellin type kernel having a fixed-point singularity at $x = y = 0$. Since the corresponding Mellin convolution operator

$$(Kf)(y) = \int_0^1 k(x,y)f(x)dx$$

is non-compact, a crucial challenge in the numerical treatment of such equations is the proof of the stability of the chosen numerical methods.

Here, we propose a Nyström method suitably modified in order to achieve the theoretical stability in the case where the kernel $k(x,y)$ in (2) satisfies the condition

$$\int_0^{+\infty} t^{-1+\sigma}|\tilde{k}(t)|dt < +\infty, \quad \sigma > 0. \tag{3}$$

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We also provide an error estimate in weighted uniform norm and prove the well-conditioning of the involved linear systems. Some numerical tests which confirm the efficiency of the method are shown.

Keywords: Mellin kernel, Mellin convolution equations, Nyström method, Gaussian rule.

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Bibliography

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