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NUMERICAL METHODS FOR BISINGULAR CAUCHY INTEGRAL EQUATIONS

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Abstract

Singular integral equations with Cauchy kernels arise in the mathematical modelling of several problems of the applied sciences like aerodynamics, elasticity, fluid flow problems and crack theory. However, although the applicative nature of these type of equations, according to our knowledge, very few numerical methods are disposable in the literature in the multivariate case.

In this talk we investigate the numerical treatment of a complete bisingular integral equation of the first kind, defined on the square $S = [-1, 1] \times [-1, 1]$ and having the following form (see [1, 2])

$$\begin{aligned}\frac{1}{\pi^2} \oint_S \frac{f(x,y)}{(x-t)(y-s)} \sqrt{\frac{1-x}{1+x}} \sqrt{\frac{1-y}{1+y}} dxdy \quad &+ \quad \int_S f(x,y)k(x,y,t,s) \sqrt{\frac{1-x}{1+x}} \sqrt{\frac{1-y}{1+y}} dxdy \\ &= \quad g(t,s), \qquad\qquad (t,s) \in S,\end{aligned}$$

where the symbol $\oint$ means that the integral has to be interpreted in the Cauchy principal value sense, f is the bivariate unknown function, g is a given right-hand side, and k is a given kernel function of four variables defined on $S \times S$.

We propose numerical methods based on the global polynomial approximation in two variables and using the zeros of the Chebyshev orthogonal polynomials of third and fourth kinds. For the proposed methods we prove the stability and the convergence in suitable Sobolev subspaces of the weighted L^2 space. The well conditioning of the involved linear systems is also showed.

Keywords: bisingular integral equations, cubature method, collocation method, Lagrange interpolation

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