

AVERAGE NUMBER OF COMPLEX ROOTS OF RANDOM SUMS

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Abstract

In 1943 Kac gave an explicit formula for the number of real roots of the random polynomial $P_n(z) = \eta_0 + \eta_1 z + \cdots + \eta_{n-1} z^{n-1}$, $z \in \mathbb{C}$, in any measurable subset of the reals, where $\eta_0, \eta_1, \dots, \eta_{n-1}$ are independent and identically distributed standard normal random variables. Kac's result was extended in 1995 by Shepp and Vanderbei, who obtained an explicit formula for the expected number of complex roots in any measurable subsets of the complex plane. Then in 2015 Vanderbei gave analogous explicit formulas for the random sum $S_n(z) = \eta_0 f_0(z) + \eta_1 f_1(z) + \cdots + \eta_{n-1} f_{n-1}(z)$, where $f_0, f_1, \dots, f_{n-1}$ are given analytic functions that are real-valued on the real line. In this talk I will present explicit formulas for the average intensities of the distribution of complex roots of both P_n and S_n when the coefficients $\eta_0, \eta_1, \dots, \eta_{n-1}$ are assumed to be independent complex Gaussian random variables, as well as numerical computations of the intensity functions and the empirical distributions for some special cases of random Weyl polynomials, random Taylor polynomials, and random truncated Fourier cosine series.

Keywords: Asymptotic formula, expected number of roots, random polynomial, random sum.

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