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ON THE CONVERGENCE OF GARGANTINI-FARMER-LOIZOU TYPE ITERATIVE METHODS FOR SIMULTANEOUS APPROXIMATION OF MULTIPLE POLYNOMIAL ZEROS

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Abstract

This talk deals with a family of high-order iterative methods for approximating all zeros (of known multiplicities) of a polynomial simultaneously.

Let $\mathbb{K}$ be a valued field, $\mathbb{K}[z]$ be the ring of polynomials over $\mathbb{K}$, and let $f \in \mathbb{K}[z]$ be a polynomial of degree $n \geq 2$ which splits in $\mathbb{K}$ and $\xi_1, \ldots, \xi_s$ be all distinct zeros of f of multiplicities $m_1, \ldots, m_s$ $(m_1 + \ldots + m_s = n)$. We define recursively a sequence $(T^{(N)})_{N=0}^{\infty}$ of functions $T^{(N)} \colon D_N \subset \mathbb{K}^s \to \mathbb{K}^s$ by setting $T^{(0)}(x) \equiv x$ and

$$T_i^{(N+1)}(x) = x_i - \frac{m_i f(x_i)}{f'(x_i) - f(x_i) \sum_{j \neq i} m_j / (x_i - T_j^{(N)}(x))} \quad (i = 1, \ldots, s).$$

For every natural number N, we define in $\mathbb{K}^s$ the following fixed-point iteration:

$$x^{(k+1)} = T^{(N)}(x^{(k)}), \qquad k = 0, 1, 2, \ldots. \tag{1}$$

In the case $N = 1$, the iterative method (1) was independently introduced by Farmer and Loizou [1] in 1977 and Gargantini [2] in 1978. The family of all iterative methods (1) is due to Kyurkchiev, Andreev and Popov [3].

We have obtained two types of local convergence theorems for the iterative methods (1) with error estimates for every $k \geq 0$. This study is a continuation of [6]. The new results are obtained by applying a new approach for convergence analysis of Picard-type iterative methods, which was proposed recently in [4].

The present talk is devoted to discussion of our local convergence result of the second type. This result generalizes a recent result of Proinov [5] for the classical Gargantini-Farmer-Loizou method, but only in the case of maximum-norm $\|\cdot\|_\infty$ in $\mathbb{K}^s$.

The following theorem is an immediate consequence of the main result of our study.

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Theorem 1. *Let $x^{(0)} \in \mathbb{K}^s$ be an initial approximation with distinct components such that*

$$\|x^{(0)} - \xi\|_\infty < \frac{2\,\delta(x^{(0)})}{3 + \sqrt{8n/m - 7}}\,, \tag{2}$$

where $\xi = (\xi_1, \ldots, \xi_s)$, $m = \min\{m_1, \ldots, m_s\}$ and the function $\delta \colon \mathbb{K}^s \to \mathbb{R}_+$ is defined by $\delta(x) = \min_{i \neq j} |x_i - x_j|$. Then the iteration (1) *converges to ξ with order $2N + 1$.*

In the very special case when $N = 1$ and f has only simple zeros, this theorem improves the classical result of Wang and Zhao [7].

Keywords: iterative methods, accelerated convergence, multiple polynomial zeros, Gargantini-Farmer-Loizou type methods, local convergence, error estimates.

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