

## RECENT PROGRESS ON THE GASCA-MAEZTU CONJECTURE

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### Abstract

An  $n$ -poised set in two dimensions is a set of nodes admitting unique bivariate interpolation with polynomials of total degree at most  $n$ . We are interested in poised sets with the property that all fundamental polynomials are products of linear factors. In 1982, M. Gasca and J. I. Maeztu [1] conjectured that every such set necessarily contains  $n + 1$  collinear nodes. Up to now, this conjecture has only been proved for  $n \leq 5$  ([2], [3], [4], [5]).

A key argument is the maximal possible number of lines containing a certain number of nodes, and the possible number of uses of such lines. e.g., a line containing only one node can not be used at all, due to the uniqueness of the fundamental polynomials; and a so-called maximal line containing  $n + 1$  nodes has to be used by all the remaining nodes. We present recent results on the number of such uses.

**Keywords:** polynomial interpolation, poised set, fundamental polynomial, maximal line, Gasca-Maeztu conjecture.

**AMS Classification:** 41A05, 41A63, 14H50.

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